The 3-dim. toric variety: $M, N$: 3-dim. dual lattices, $T = (\mathbb{C}^*)^3$. 
$\Delta \subset M \otimes_{\mathbb{Z}} \mathbb{R}$ a 3-dimensional lattice polytope, 
$\Sigma_{\Delta}$: The normal fan of $\Delta$, with rays $\Sigma_{\Delta}[1]$. 
$\mathbb{P}_{\Delta}$: The toric variety to the polytope $\Delta$ via the normal fan. $\mathbb{P}_{\Sigma}$: The toric variety to the fan $\Sigma$.

The hypersurface: 
$f = \sum_{m \in \Delta \cap M} a_m x^m$, $a_m \in \mathbb{C}$ 
a Laurent polyn. with Newton polytope $\Delta$. For a 3-dimensional polytope $Z$ 
$Z_f := \{ f = 0 \} \subset T$, the associated hypersurface. For a 3-dimensional polytope $P$ we write $Z_P$ for the closure of $Z_f$ in the toric variety $\mathbb{P}_P$.

The nondegeneracy condition: 
The hypersurface should intersect the toric strata transversally.

The Fine interior and the canonical closure: 
For $\nu \in N$ let 
$\text{ord}_{\Delta}(\nu) := \min_{m \in \Delta \cap M} \langle m, \nu \rangle$. 
Then define the Fine interior 
$F(\Delta) := \{ x \in M_{\mathbb{R}} \mid \langle x, \nu \rangle \geq \text{ord}_{\Delta}(\nu) + 1, \nu \in N \setminus \{0\} \}$. 
The support $S_F(\Delta)$ are the $\nu \in N \setminus \{0\}$ with 
$\text{ord}_{F(\Delta)}(\nu) = \text{ord}_{\Delta}(\nu) + 1$.

$C(\Delta) := \{ x \in M_{\mathbb{R}} \mid \langle x, \nu \rangle \geq \text{ord}_{\Delta}(\nu) \forall \nu \in S_F(\Delta) \}$ is called the canonical closure of $\Delta$. We obtain $F(\Delta) \subset \Delta \subset C(\Delta)$.

Classification of some polytopes: 
$\Delta$ is called: 
- canonical, if it contains just 0 in its interior.
- Fano, if its vertices are primitive points.
There are just 49 3-dim. canonical Fano polytopes left with $\dim F(\Delta) = 3$.

Result: Among the 49 polytopes there are just 5 iso. types for the Fine interior $F(\Delta)$.

Result: For $F$ out of these 5 types there is exactly one maximal polytope $\Delta$ with $F(\Delta) = F$.

Result: For $\Delta$ a maximal polytope: 
$\mathbb{P}_{\Delta} \cong \mathbb{P}_P \cong \mathbb{P}_C(\Delta) \cong \mathbb{P}_{F(\Delta)}$.

Constructing canonical/minimal models: 
Result (Bat20): If $k := \dim F(\Delta) \geq 0$, the Kodaira dimension of $Z_P$ equals 
$\kappa(Z_P) = \min(k, 2)$. For our 49 polytopes $\Delta$ we get $\kappa(Z_P) = 2$.

Result (Bat20): $Z_P$ has at most canonical sing. and $K_{Z_P}$ is nef. The closure $Z_{F(\Delta)}$ gets a canonical model.

Result (Bat20): $\Sigma_P[1] \subset S_F(\Delta)$. Choose a refinement $\Sigma$ of $\Sigma_P$ with $\Sigma[1] = S_F(\Delta)$, then the closure of $Z_f$ in $\mathbb{P}_\Sigma$ gets a minimal model.

Result (Gie21): In 46 cases $Z_{F(\Delta)}$ gets a 
Kanev surface, i.e. $p_g(Z_{F(\Delta)}) = 1$, $K^2_{Z_{F(\Delta)}} = 1$ and in 3 cases a surface of Todorov type, that is 
$p_g(Z_{F(\Delta)}) = 1$, $q(Z_{F(\Delta)}) = 0$, $K^2_{Z_{F(\Delta)}} = 2$.

Illustration of some polytopes: 
3 polytopes with the left one maximal.

A cross-section of the only 3-dim. cone that gets refined in going from $\Sigma_{F(\Delta)}$ over $\Sigma_P$ to $\Sigma$. The subdivision defines $\Sigma_P$ and the additional rays are vectors defining $\Sigma$. These are all support vectors we need for constructing a minimal model.

An Interpretation and further results (Gie21): 
Result: The sing. of $Z_P$ are of type $A_k$. The interior points in the cross-section define the Dynkin diagram of the sing. of $Z_{F(\Delta)}$ at $p$. So $Z_{F(\Delta)}$ has an $A_2$ (an $E_6$) singularity in the middle (right) picture.

Result: This subdivision yields a degen. of $Z_{F(\Delta)}$ into two weak del Pezzo surfaces. By this the generic Picard number of several Kanev surfaces are computable.

References: (Bat20): Canonical models of toric hypersurfaces. (Gie21): Kanev surfaces in toric 3-folds.