## Concluding Report on Chinedu Izuchukwu's Research Stay at the Department of Mathematical Sciences, NTNU, Trondheim

**Duration of Stay:** December 12, 2024 – January 12, 2025 **Purpose of Visit:** Research on Vector Variational Inequalities

Dr. Chinedu Izuchukwu's research visit to the Department of Mathematical Sciences, NTNU, Trondheim, from December 12, 2024, to January 12, 2025, was dedicated to advancing the field of vector variational inequalities (VVIs). This fundamental area of optimization theory addresses both scalar and vector optimization problems and has numerous applications, including traffic flow modeling, uncertain optimization, machine learning, artificial intelligence, and image processing. During this visit, Dr. Izuchukwu focused on developing computationally efficient and broadly applicable methods for solving VVIs and studying their convergence properties, numerical behavior, and practical implementation in real-world problems. The collaboration has yielded substantial progress, laying the groundwork for further advancements in this promising area of research.

During the visit, Dr. Izuchukwu engaged in extensive discussions on advancing techniques for solving optimization problems. He showed particular interest in second-order dynamical systems for optimization—a technique primarily applied to scalar optimization problems. Dr. Izuchukwu expressed a strong determination to extend this approach to vector optimization and vector variational inequalities, despite the significant challenges posed by the complexity of such problems. Collaborative discussions explored potential strategies for tackling these challenges, including the use of scalarization techniques. These fruitful exchanges have identified promising research directions, and we plan to continue pursuing these ideas beyond the visit.

Dr. Izuchukwu delivered a lecture titled "Advances in Relaxed Inertial Algorithms for Monotone Inclusion Problems and Their Applications." Monotone inclusion problems, which involve finding zeros of the sum of monotone operators, are foundational in optimization, variational analysis, machine learning, and applied mathematics. His presentation explored the development and analysis of relaxed inertial splitting algorithms for solving monotone inclusion problems, with discussions on the role of resolvent operators, novel inertial and relaxation parameter constructions, and their implications. Applications were highlighted in machine learning, signal processing, and traffic equilibrium modeling, with particular attention to the theoretical foundations and practical advantages of these algorithms. The lecture topics closely aligned with Dr. Izuchukwu's research interests in solving vector optimization problems via second-order dynamical systems and sparked lively discussions among attendees.

The research collaboration during Dr. Izuchukwu's visit centered on constructing faster, computationally inexpensive methods to solve VVIs, outperforming existing techniques in the literature. Efforts were directed toward real-world applications, such as optimizing traffic networks to reduce congestion, and exploring VVIs' capacity to address uncertainties in vector-valued optimization problems. The collaboration also focused on analyzing the convergence rates, stability, and error estimates of the proposed methods. Regular brainstorming sessions enriched the research outcomes and inspired new ideas for future exploration.

Dr. Izuchukwu's research bridges mathematics, computational science, and real-world applications. His primary interest lies in vector variational inequalities, which provide a robust framework for solving complex optimization problems across domains like transportation, artificial intelligence, and economics. His approachable style and clear explanations during his lecture and informal discussions made advanced topics accessible to a broad audience, fostering enthusiasm and stimulating further research directions.

The outcomes of this research visit include a comprehensive study of VVIs with applications to traffic flow models and uncertain optimization problems. Two high-quality papers are being prepared for submission to **SIAM Journal on Optimization** and **Mathematical Programming**, and the findings will be presented at international conferences and workshops. Additionally, a long-term research collaboration network has been established, ensuring continued progress in this field. Dr. Izuchukwu's visit has significantly advanced our research agenda and reinforced the IMF's commitment to fostering innovation and interdisciplinary collaboration. His contributions have laid a strong foundation for future advancements in optimization and its applications.